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Estimating strain from fault slip using a line sample

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Abstract—A method is presented that enables data for faults with different orientations and displacements, measured along a single straight line, to be used to estimate the magnitudes and orientations of the principal strain axes. The method combines two well-established techniques. When sampling along a line, the probability of intersecting a fault is affected by its orientation. This sampling bias may be minimized by the use of a weighting, $w = 1/\cos\gamma$, where γ = angle between the perpendicular to the fault and the sample line. The displacement gradient and Lagrangian strain tensors may then be used to describe the deformation with respect to the undeformed state. The method can also be applied to such structures as veins and stylolites. As an example of the use of the method, 1340 normal faults have been measured along a 6 km length of Cretaceous chalk cliffs at Flamborough Head, Humberside, U.K. Consistent strain estimates have been obtained for different portions of the cliff.

INTRODUCTION

The discontinuous deformation produced by displacements on faults can be summed to provide an estimate of strain, provided that the fault spacings and displacements are small relative to the size of the area under investigation (Jamison 1989, Wojtal 1989). The term *strain* should only be applied to continuous deformation, hence Jamison (1989) introduced the term *fault strain* to describe the deformation caused by a number of faults. Jamison (1989), Wotjal (1989) and Marrett & Allmendinger (1990) give applications of displacement gradient tensors for field data, which are analogous to the geometric and seismic moments applied to seismological problems (Molnar 1983, Marrett & Allmendinger 1990).

Molnar (1983) defines an *asymmetric moment* tensor, thus:

$$\mathbf{D}_{ij} = (sa/v)\mathbf{u}_i\mathbf{n}_j, \tag{1}$$

where s is the mean slip on the fault, a the area of the fault, v the volume of the region and $\mathbf{u}_i$, $\mathbf{n}_j$ are unit vectors parallel to the slip direction and the perpendicular to the fault plane, respectively. Equation (1) is equivalent to the displacement-gradient tensor of Jamison (1989) and of Marrett & Allmendinger (1990). For small strains, the displacement-gradient tensor is related to the Lagrangian strain tensor, $\mathbf{e}_{ij}$, thus:

$$\mathbf{e}_{ij} = \tfrac{1}{2}(\mathbf{D}_{ij} + \mathbf{D}_{ji}) = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \tag{2}$$

(Molnar 1983, Jamison 1989). $\mathbf{e}_{ij}$ is a symmetrical matrix, i.e. $x_{ij} = x_{ji}$. The total strain can be found from $\mathbf{D}_{ij}$ by using $\mathbf{e}_{ij}$ as shown in equation (2). The Lagrangian strain tensor describes strain as a function of displacement gradients in the undeformed state. The magnitudes and orientations of the principal strains (e_1, e_2 and e_3) can be found from the eigenvalues and eigenvectors of this tensor or by algebraic solution of the cubic characteristic equation.

A major problem in determining fault strain from geological, rather than seismological, data is that it is usually difficult to determine the orientations and displacements of all the faults within a three-dimensional rock mass. Random sampling of the fault population can give estimates of the orientations and relative magnitudes of the strains, but not their absolute values. Random sampling of faults in a three-dimensional rock mass is, however, almost as difficult to achieve as sampling of the entire population. Complete sampling is generally restricted to surfaces (e.g. maps or seismic sections) or line transects through a rock mass (e.g. along traverses or in drill-holes). The aim of this paper is to present a modification of the method of Jamison (1989) which allows estimates of strain to be made from data collected along a single straight line. This modified method can be used to analyse large quantities of data by the use of a simple computer program. As an example, strain estimates are made for faults in the Cretaceous chalk of Flamborough Head, Humberside.

Methods of strain analysis have advantages over 'palaeostress' orientation methods (e.g. Angelier 1984), many of which have been discussed by Marrett & Allmendinger (1990). One of the advantages of methods which use displacements to determine strains, even if produced by superposed deformation events, is that the resulting total strain still has some meaning. Palaeostress orientation methods of the type discussed by Angelier (1984) are only valid for individual stress fields and require additional constraints to separate superposed events.

METHOD FOR ESTIMATING STRAIN FROM FAULTS

Fracture data collected along a single straight line provide a biased sample of the population (Fig. 1). The probability of faults of a particular orientation inter-

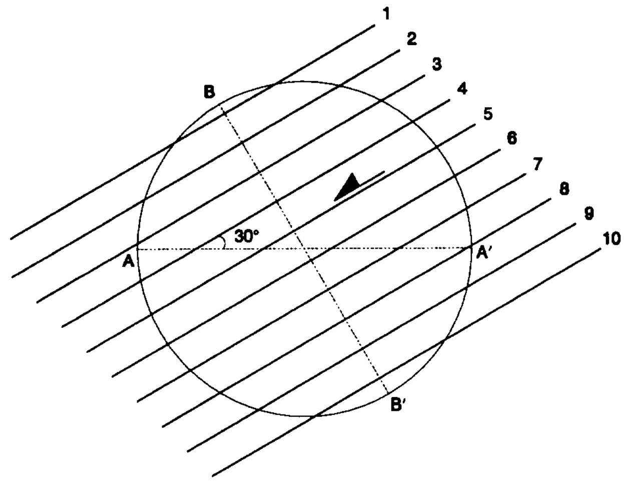

Fig. 1. A simple case (in two dimensions) of a set of parallel faults (numbered 1–10) striking at 30° to the sample line (A–A'), which forms the diameter of the volume under investigation. Five faults (4–8) are intersected by A–A', but the area under investigation is intersected by 10 faults (as shown by line B–B'). This is correctly predicted by the weighting (1/cos γ, where γ = 60°). The weighting is, therefore, essential to predict correct strains from line samples (see text for details).

secting a given line is determined by the angle between the faults and the line and by the fracture density (fault area per unit volume). Because it is the fracture density and the displacements which are needed to calculate strain, the orientation bias can be removed by *weighting* or *correcting* the data by a factor $w = 1/\cos\gamma$, where γ is the angle between the sample line and the perpendicular to each fracture. Similar techniques are widely used to determine fracture spacing (e.g. LaPointe & Hudson 1985, Barton & Zoback 1990, 1992).

The displacement-gradient tensor can be weighted to become:

$$\mathbf{D}_{ij} = \frac{ws}{d}\begin{bmatrix} n_1u_1 & n_1u_2 & n_1u_3 \\ n_2u_1 & n_2u_2 & n_2u_3 \\ n_3u_1 & n_3u_2 & n_3u_3 \end{bmatrix}, \tag{3}$$

where w is the weighting factor, s is the displacement of each fault and d is the length of the sample line. The displacement gradient tensor presented in equation (3) is equivalent to that presented in equation (1) and in equation (A8) of Jamison (1989), except that the weighting factor (w) has been introduced. Total strain can be estimated by summing $\mathbf{D}_{ij}$ and $\mathbf{D}_{ji}$ for all of the faults along the straight line sample. Thus, the strains produced by faults of different orientations and displacements which intersect the line sample can be added to estimate the strain ellipsoid.

The unit vectors $\mathbf{n} = (n_1\, n_2\, n_3)$ and $\mathbf{u} = (u_1\, u_2\, u_3)$ are normal to the fault plane and parallel to the displacement direction, respectively. Vectors are measured in relation to geographical reference axes (1, 2, 3) which are oriented north, east and vertically down, respectively. Where the fault dips θ towards ϕ, and the displacement direction plunges ξ towards δ, then:

$$\mathbf{n} = (-\cos\phi\sin\theta, \quad -\sin\phi\sin\theta, \quad \cos\theta)$$

$$\mathbf{u} = (\cos\delta\cos\xi, \quad \sin\delta\cos\xi, \quad \sin\xi).$$

Since, by convention, $\mathbf{n}$ is directed downward it can be regarded as a unit vector directed into the footwall of the fault. The unit vector $\mathbf{u}$ is then the displacement direction of this footwall block, relative to the hanging-wall, and is directed upward (u_3 is negative) for normal faults and downward (u_3 is positive) for reverse faults. This sign convention is important because it ensures compatible assignment of senses to the displacement vector and results in strains which are positive for lengthening and negative for shortening, as is conventional in tectonics. The technique can be extended to veins and dykes by using a displacement vector $\mathbf{u} = \mathbf{n}$ and to solution seams using $\mathbf{u} = -\mathbf{n}$, where displacement is perpendicular to the planar structure.

The angle, γ, between the sample line and the perpendicular to each fault can be found from the scalar (or dot) product of unit vectors $\mathbf{n}$ and $\mathbf{t}$, where $\mathbf{t}$ is parallel to the sample line, thus:

$$\cos\gamma = \mathbf{n}\cdot\mathbf{t} = (n_1t_1 + n_2t_2 + n_3t_3). \tag{4}$$

As the angle, γ, increases (i.e. when the sample line is at a low angle to the fault), the weighting factor (1/cos γ) increases; the weighting tends to ∞ as γ approaches 90°. Clearly the tensor $\mathbf{D}_{ij}$ becomes unstable with such large weighting factors. To avoid this, two procedures can be used. First, sample lines should be chosen at high angles to faults, such that γ is $\ll$90°. If necessary, more than one sample line could be used for different fault sets. Second, where it is not possible to choose a sample line at a high angle to all of the faults, an arbitrary maximum weighting factor, w, can be chosen. The maximum w value used in this paper is 4, corresponding to $\gamma \approx 75°$; faults with $\gamma > 75°$ are assigned $w = 4$.

THE IMPORTANCE OF THE WEIGHTING FACTOR

Before presenting a field example, two simple cases will be considered in order to illustrate the necessity of applying the orientation to weighting fault data. In Fig. 1, a set of faults have the same orientation, spacing and displacement. Consider a sample line A–A', which is 100 m long and is at 30° to the five faults it intersects, each of which has 1 m displacement, of the sense shown. If strain is estimated from line A–A' without using the weighting (w), then e_1 is 2.5% and 15° clockwise of A–A', whilst e_3 is −2.5% and 105° clockwise of A–A'. If the weighting is used, e_1 and e_3 have the same orientations as the unweighted estimates, but are 5 and −5%, respectively. The same result as the weighted estimate for A–A' is obtained when using line B–B', which intersects all of the faults normal to their planes, which is exactly what the weighting factor is designed to achieve. Note that the deformation is a simple shear with a shear strain of 0.1, which yields principal extensions of ±5%.

Now consider a conjugate pair of normal faults, each dipping at 60° and with a displacement of 1 m, measured along a 100 m horizontal traverse perpendicular to their strike (Fig. 2). Note that each fault contributes a hori-

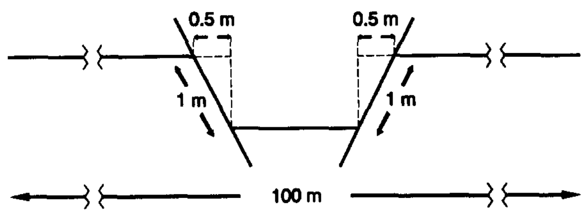

Fig. 2. Simple case of two conjugate extensional faults which are perpendicular to the cross-section. They extend the line sample by 1% (see text for details).

zontal extension (heave) of 0.5 m (i.e. 1 m × cos 60°), making a total extension of 1 m in 100 m; $e_1 = +0.01$. If the strain is calculated without weighting (i.e. $w = 1$ in equation 3), a horizontal extension of +0.0087 is found, which does not agree with the true extension. Since each fault is at 60° to the sample line, $\gamma = \pm 30°$ and $w = 1/\cos 30° = 1.155$. Using this weighting in equation (3), yields the correct extension (e_1) of +0.01.

Although the two examples are simple and have solutions which are mathematically trivial (i.e. obvious without recourse to the displacement-gradient tensor and geometric moments), they clearly illustrate the importance of using the weighting factor when estimating strain from line samples. Note that the weighting is applied to each fault and works where the faults have different orientations, spacings and displacements. It should also be pointed out that planar samples (e.g. from maps, seismic sections, etc.) obviously require a similar correction where the faults are not normal to the plane.

EXAMPLE OF NORMAL FAULTS AT FLAMBOROUGH HEAD

To illustrate the method, an analysis has been made of 1340 normal faults in Upper Cretaceous chalk cliffs along a 6 km line traverse on the south side of Flamborough Head, Humberside, U.K. Some of the faults have an oblique-slip component, and they have displacements of up to 6 m (Peacock & Sanderson 1992, in press). The cliff is approximately straight and the quality of the exposure is very good. The upper part of the cliff is inaccessible and fault displacements are difficult to identify on the wave-cut platform. The study is therefore limited to a single approximately straight line, at the base of the cliff. This sampling line has been divided into three approximately WSW-trending portions, separated by short breaks in exposure. The position, orientation and displacement of all faults which intersect the sample line were measured.

A summary of the data and derived strain estimates are shown in Table 1. The amount of strain varies along the cliff section (Peacock & Sanderson in press), so the results are averaged over the lengths of the three individual sample lines. The weighted estimates of strain are approximately twice as large as the unweighted measurements, with the e_2 estimates showing particularly large increases. e_3 is consistently sub-vertical and indicates approximately 2% shortening for weighted estimates. e_1 and e_2 are consistently sub-horizontal and are similar in magnitude for weighted estimates, at about 1% extension. This indicates approximately equal extension in all horizontal directions. Unweighted e_1 orientations are consistently orientated approximately NE–SW, which is a reflection of under-sampling of faults at a low angle to the line of sampling. The weighted estimates for e_1 are also orientated approximately NE–SW for the Sewerby to Dykes End and South Landing to High Stacks portions of the sampling line. The Dykes End to South Landing portion, however, shows a weighted e_1 orientation of approximately NNW–SSE. In spite of the differences between e_1 orientations, the method produces similar strain estimates for the three portions of the sample line, with $e_1 \approx e_2$.

Table 1. Unweighted strain measurements and weighted strain estimates for three approximately straight sample lines: Sewerby (grid reference TA20166866) to Dykes End (TA21596918); Dykes End to South Landing (TA23126924); and South Landing to High Stacks (TA25787041). The Dykes End to South Landing portion includes a 90 m break caused by a landslide. The orientations of the strain axes are shown as dips and dip directions, e.g. the weighted e_3 axis for Sewerby to Dykes End dips at 84.3° towards an azimuth of 357°

	Sewerby to Dykes End	Dykes End to South Landing	South Landing to High Stacks
Section length	1449 m	1360 m	2891 m
Line orientation	071°	088°	063°
Number of faults	384	396	560
Extension along sample line	15.88 m	17.192 m	19.404 m
	(1.096%)	(1.264%)	(0.671%)
e_1 (without weighting)	0.00798	0.00852	0.00452
	1.1° to 252.7°	2.1° to 268.4°	0.8° to 238°
e_2 (without weighting)	0.00335	0.00657	0.00287
	3.9° to 162.6°	5.4° to 178.2°	7.4° to 147.9°
e_3 (without weighting)	−0.001133	−0.01509	−0.0074
	86° to 358.8°	84.2° to 019.2°	82.5° to 334.4°
e_1 (weighted)	0.01095	0.01722	0.00673
	0.8° to 259.1°	4.8° to 163.1°	1° to 076.5°
e_2 (weighted)	0.0086	0.01229	0.00623
	5.6° to 169°	3.1° to 253.4°	8.1° to 166.6°
e_3 (weighted)	−0.01955	−0.0295	−0.01295
	84.3° to 357°	84.3° to 016.4°	81.8° to 339.4°

DISCUSSION OF SAMPLING ERRORS AND PROCEDURES

As stated at the start of the Introduction, strain can be estimated from faults provided the spacings and displacements are small relative to the size of the area under investigation. The accuracy of the method is improved as the number of faults (or the length of the line sample) is increased.

Fault populations have commonly been found to show power-law distributions (e.g. Scholz & Cowie 1990, Walsh *et al.* 1991, Marrett & Allmendinger 1992). In practice it is only possible to resolve faults at Flamborough Head with a displacement of 30 mm–10 m; it is probable that many smaller faults exist. Larger faults, possibly separating or bounding the sections, may also exist and contribute to the extension. Thus the derived estimates of strain, whilst minimizing sampling effects caused by fault orientation, must be interpreted carefully within their geological context.

The method could be used to combine strains associated with different scales of structures, such as faults, veins and pressure solution seams. Since an average strain is obtained along a line, sample lengths should be chosen to cover statistically homogeneous regions. If there are large variations in strain along the sample lines, they should be sub-divided; Wojtal (1989) provides some discussion of this. Similarly, lines from differently deformed regions should not be combined.

The method of strain measurement described by Marrett & Allmendinger (1990) requires the mean displacement on a fault plane to be estimated. As shown by Marrett & Allmendinger (1990, fig. 3), the estimate of mean displacement depends upon such factors as the displacement–distance characteristics of the fault. Obviously, line samples do not intersect most faults near a point of mean displacement. It is probable, however, that the randomly measured displacements from a large number of sampled faults approximates the mean displacements of the faults (see Marrett & Allmendinger 1990 for discussion).

The method presented in this paper gives an estimate of strain from data collected along a single straight line. Many practical situations limit sampling to such circumstances and the method is clearly applicable to sampling in drill-holes, mines and tunnels, or where data are restricted to a single section through a bed. Where sampling is less restricted, data from different traverses through a region can be combined by addition of the deformation gradient tensors. Where fractures have very variable orientations, it is best to use several lines with different directions, where that is possible. The method is not an excuse for bad sampling practice.

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